



# Maths Points

Junior and Leaving Cert

2019 PAPER 1 – Q8

LEAVING CERT HIGHER LEVEL





## Section B

Question 8 (a)

Question 8 (b)

Question 8 (c)

Question 8 (d)

Question 8 (e)

The weekly revenue produced by a company manufacturing air conditioning units is seasonal. The revenue (in euro) can be approximated by the function:

$$r(t) = 22\,500 \cos\left(\frac{\pi}{26}t\right) + 37\,500, \quad t \geq 0$$

where  $t$  is the number of weeks measured from the beginning of July and  $\left(\frac{\pi}{26}t\right)$  is in radians.

Find the approximate revenue produced 20 weeks after the beginning of July. Give your answer correct to the nearest euro.



$$r(t) = 22\,500 \cos\left(\frac{\pi}{26}t\right) + 37\,500$$

$$r(t) = 22\,500 \cos\left(\frac{\pi}{26}t\right) + 37\,500$$

$$r(20) = 22\,500 \cos\left(\frac{\pi}{26}(20)\right) + 37\,500 \quad \leftarrow t = 20$$

$$r(20) = 20,658.51$$

$$r(20) \approx \text{€}20,659$$

Make sure your calculator is in radian mode!

The revenue after 20 weeks is approximately €20,659.



Find the two values of the time  $t$ , within the first 52 weeks, when the revenue is approximately €26 250.



$$r(t) = 22\,500 \cos\left(\frac{\pi}{26}t\right) + 37\,500$$

Let the formula equal €26,250 and solve for the time,  $t$ .

$$22\,500 \cos\left(\frac{\pi}{26}t\right) + 37\,500 = 26\,250$$

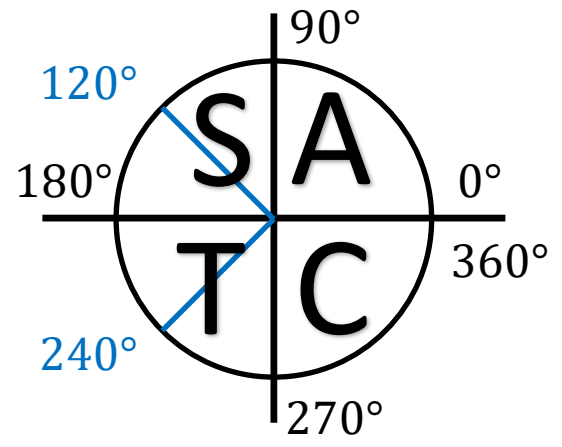
$$22\,500 \cos\left(\frac{\pi}{26}t\right) = -11\,250$$

$$\cos\left(\frac{\pi}{26}t\right) = -\frac{1}{2}$$

$$\frac{\pi}{26}t = \cos^{-1}\left(-\frac{1}{2}\right)$$

**Reference Angle**  
 $\cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$

Use CAST method to select quadrant.  
 cos is negative in the 2<sup>nd</sup> and 3<sup>rd</sup> quadrants.



$$t = 120^\circ \text{ \& \ } 240^\circ$$

$$t = \frac{2\pi}{3} \text{ \& \ } \frac{4\pi}{3}$$

Convert to radians.  
 Multiply by  $\frac{\pi}{180}$ .

$$\frac{\pi}{26}t = \frac{2\pi}{3}$$

$$t = \frac{52}{3}$$

$$\frac{\pi}{26}t = \frac{4\pi}{3}$$

$$t = \frac{104}{3}$$



Find  $r'(t)$ , the derivative of  $r(t) = 22\,500 \cos\left(\frac{\pi}{26}t\right) + 37\,500$ .

$$r(t) = 22\,500 \cos\left(\frac{\pi}{26}t\right) + 37\,500$$

$$r(t) = 22\,500 \cos\left(\frac{\pi}{26}t\right) + 37\,500$$

$$r'(t) = -22\,500 \sin\left(\frac{\pi}{26}t\right) \left(\frac{\pi}{26}\right)$$

$$r'(t) = -\frac{11\,250\pi}{13} \sin\left(\frac{\pi}{26}t\right)$$

$$f(x) = \cos u$$

$$f'(x) = -\sin u \left(\frac{du}{dx}\right)$$



Use calculus to show that the revenue is increasing 30 weeks after the beginning of July.

$$r(t) = 22\,500 \cos\left(\frac{\pi}{26}t\right) + 37\,500$$

$$r'(t) = -\frac{11\,250\pi}{13} \sin\left(\frac{\pi}{26}t\right)$$

A function  $r(t)$  is increasing if  $r'(t) > 0$ .

$$r'(t) = -\frac{11\,250\pi}{13} \sin\left(\frac{\pi}{26}t\right)$$

$$r'(30) = -\frac{11\,250\pi}{13} \sin\left(\frac{\pi}{26}(30)\right) \quad \leftarrow t = 30$$

$$r'(30) = 1263.44$$

$1263.44 > 0$  therefore the revenue is increasing 30 weeks after the beginning of July.



Find a value for the time  $t$ , within the first 52 weeks, when the revenue is at a minimum.  
Use  $r''(t)$ , to verify your answer.



$$r(t) = 22\,500 \cos\left(\frac{\pi}{26}t\right) + 37\,500$$

$$r'(t) = -\frac{11\,250\pi}{13} \sin\left(\frac{\pi}{26}t\right)$$

$$-\frac{11\,250\pi}{13} \sin\left(\frac{\pi}{26}t\right) = 0$$

$$\sin\left(\frac{\pi}{26}t\right) = 0$$

$$\frac{\pi}{26}t = \sin^{-1}(0)$$

$$\frac{\pi}{26}t = 0$$

$$t = 0$$

$$\frac{\pi}{26}t = \pi$$

$$t = 26$$

**Max/Min**  
 $r'(t) = 0$   
and solve for  $t$

Unit Circle

See next slide.

**2<sup>nd</sup> Derivative Test**  
 $r''(t) > 0$  minimum  
 $r''(t) < 0$  maximum

$$r'(t) = -\frac{11\,250\pi}{13} \sin\left(\frac{\pi}{26}t\right)$$

$$r''(t) = -\frac{11\,250\pi}{13} \cos\left(\frac{\pi}{26}t\right) \cdot \left(\frac{\pi}{26}\right)$$

$$r''(t) = -\frac{5625\pi^2}{169} \cos\left(\frac{\pi}{26}t\right)$$

$$t = 0$$

$$r''(0) = -\frac{5625\pi^2}{169} \cos\left(\frac{\pi}{26}(0)\right) = -\frac{5625\pi^2}{169}$$

$r''(0) < 0$ , maximum at 0 Weeks

$$t = 26$$

$$r''(26) = -\frac{5625\pi^2}{169} \cos\left(\frac{\pi}{26}(26)\right) = \frac{5625\pi^2}{169}$$

$r''(26) > 0$ , minimum at 26 Weeks



Find a value for the time  $t$ , within the first 52 weeks, when the revenue is at a minimum.

Use  $r''(t)$ , to verify your answer.

$$\sin\left(\frac{\pi}{26}t\right) = 0$$

$$\sin\left(\frac{\pi}{26}t\right) = 0$$

$$\frac{\pi}{26}t = \sin^{-1}(0)$$

$$\frac{\pi}{26}t = 0$$

$$t = 0$$

$$\frac{\pi}{26}t = \pi$$

$$t = 26$$

$$\frac{\pi}{26}t = 2\pi$$

$$t = 52$$

For values of cos and sin equal to 0, 1 and  $-1$  use this Unit Circle.

