Maths Points

Junior and Leaving Cert

2019 PAPER 1 – Q7

Leaving Cert Ordinary Level

2019 LCOL Paper 1 – Table of Contents

A camogie goalkeeper, on a level pitch, hit a ball straight up into the air.
The path that the ball travelled can be modelled by the function:

$$f(t) = -4t^2 + 16t + 1, \qquad t \in \mathbb{R}.$$

where t is the time, in seconds, from when the ball is hit and $f(t)$ was the height of the ball, in metres, above the pitch. The ball landed on the ground without being hit again.

At what height was the ball when it was hit by the goalkeeper?

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The ball was hit by the goalkeeper at the time, $t = 0$.

$f(t) = -4t^2 + 16t + 1$

$$f(t) = -4t^2 + 16t + 1$$
$$f(0) = -4(0)^2 + 16(0) + 1 \quad \leftarrow t = 0$$
$$f(0) = 0 + 0 + 1$$
$$f(0) = 1$$

The ball was a height of 1 m when it was hit by the goalkeeper.

Complete the table below to show the height of the ball at various intervals during the first 4 seconds of its flight.

$$f(t) = -4t^2 + 16t + 1$$

Time (t)	0	0·5	1	1·5	2	2·5	3	3·5	4
Height (m)	1	8	13	16	17	16	13	8	1

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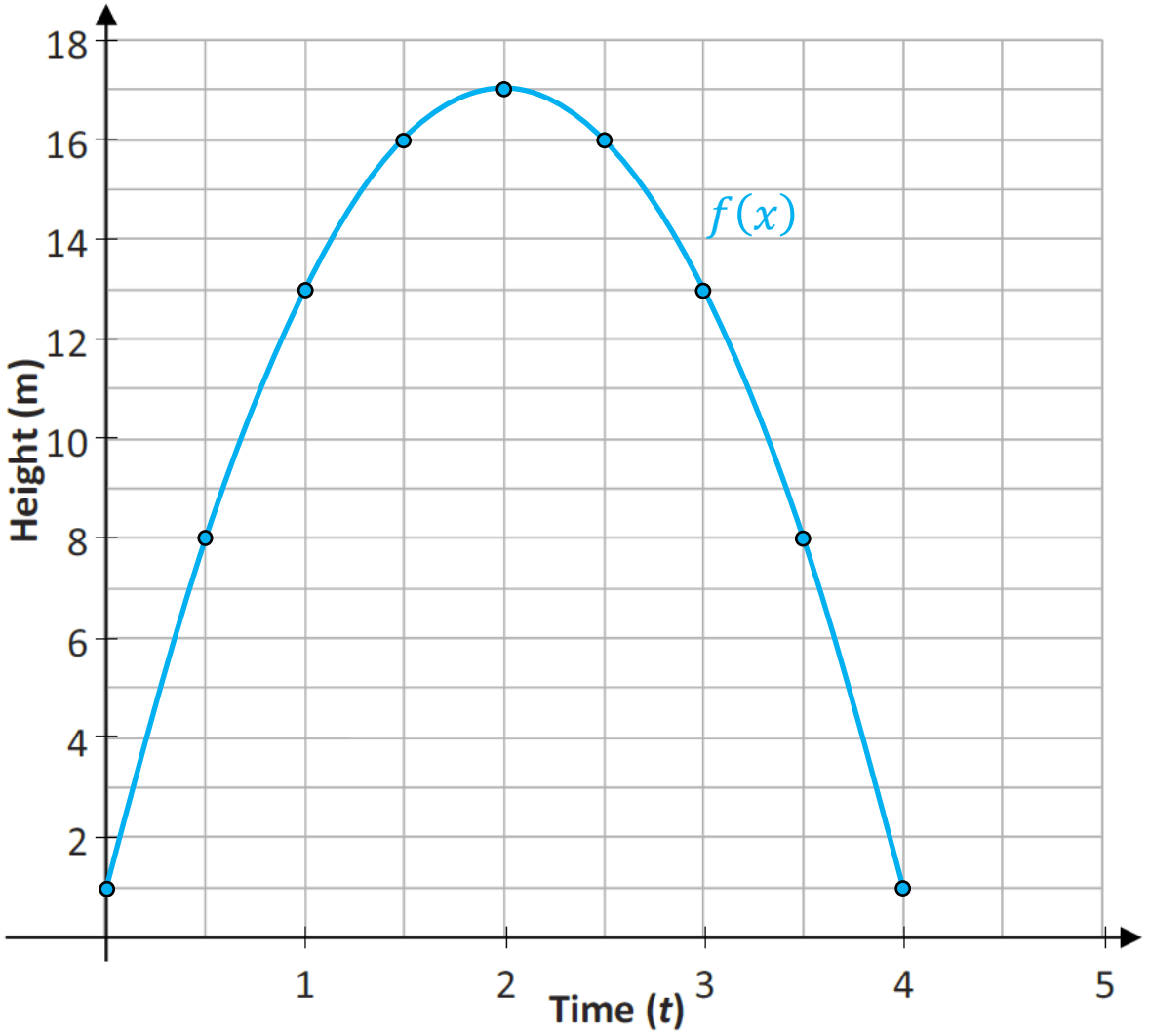
Always fill out a table.

x	$f(t) = -4t^2 + 16t + 1$	y
0	$-4(0)^2 + 16(0) + 1$	1
0.5	$-4(0.5)^2 + 16(0.5) + 1$	8
1	$-4(1)^2 + 16(1) + 1$	13
1.5	$-4(1.5)^2 + 16(1.5) + 1$	16
2	$-4(2)^2 + 16(2) + 1$	17
2.5	$-4(2.5)^2 + 16(2.5) + 1$	16
3	$-4(3)^2 + 16(3) + 1$	13
3.5	$-4(3.5)^2 + 16(3.5) + 1$	8
4	$-4(4)^2 + 16(4) + 1$	1

Press MENU and select 3.

… always check solution by using **table mode** of the calculator!

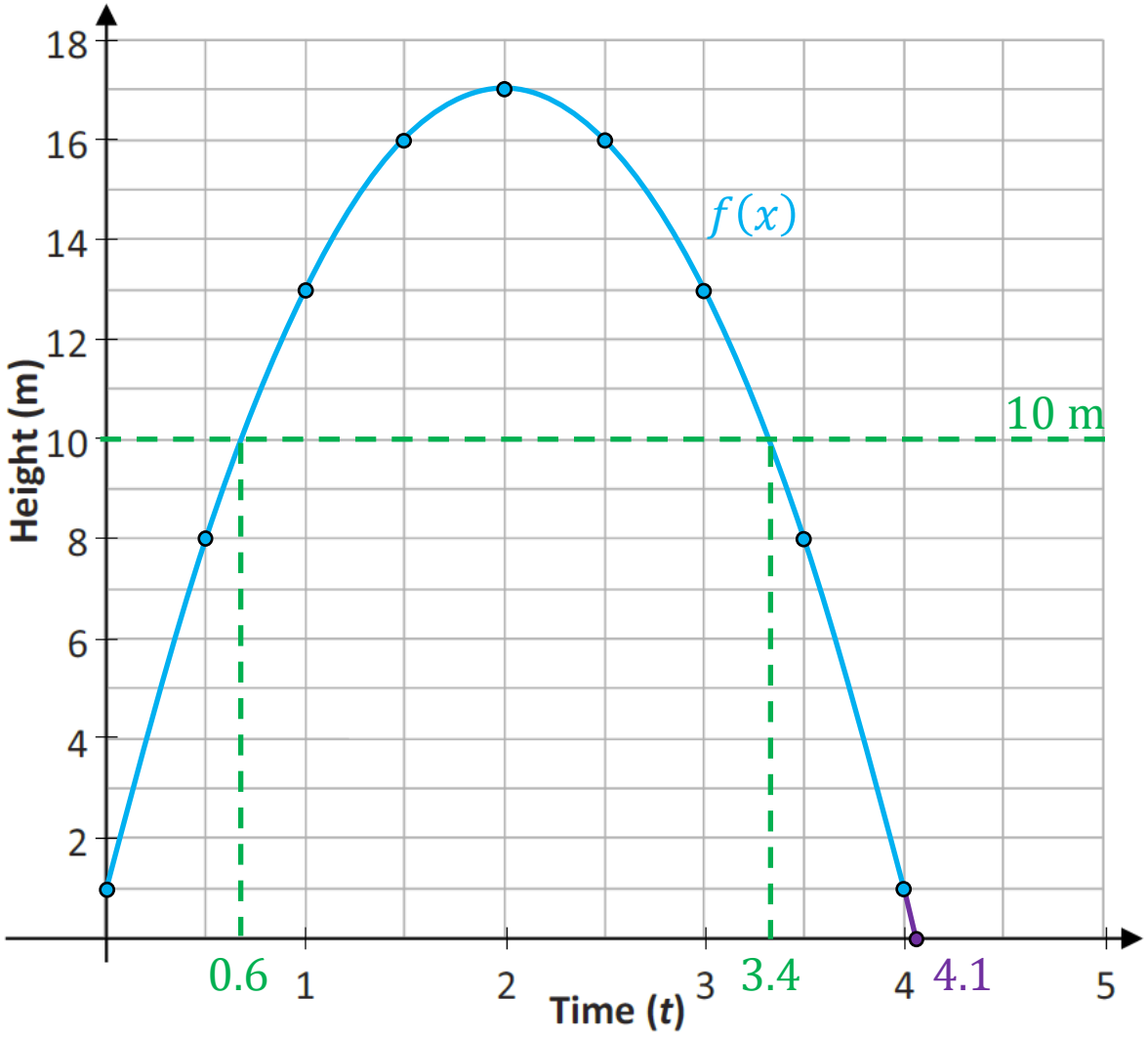
On the grid below draw a graph to show the height of the ball while it was in the air.



x	y
0	1
0.5	8
1	13
1.5	16
2	17
2.5	16
3	13
3.5	8
4	1

Use your graph to estimate:
(Show your work on the graph above)

the length of time the ball was in the air from the time it was hit until it landed on the ground



4.1 seconds

(ii)

the length of time the ball was 10 m, or more, above the ground.

The ball was 10m or more above the ground from 0.6 seconds to 3.4 seconds.

$3.4 - 0.6 = 2.8$ seconds

Find $f'(t)$, the derivative of $f(t) = -4t^2 + 16t + 1$.

$f(t) = -4t^2 + 16t + 1$

$f(t) = -4t^2 + 16t + 1$

$f'(t) = -8t + 16$

Differentiation – Power Rule

$f(x) = x^n$

$f'(x) = nx^{n-1}$

For each term bring the power to the front (multiply) and reduce the power by 1.

The derivative of any constant (a term independent of x) is 0.

Use your answer from **part (d)(i)** to find the speed of the ball when it had been in the air for 4 seconds.

Give your answer in metres per second.

$f(t) = -4t^2 + 16t + 1$

$f(t) = -4t^2 + 16t + 1$

$f'(t) = -8t + 16$

When you differentiate a function you get the **rate of change** of that function.

Speed is the change in height with respect to time!

$f'(4) = -8(4) + 16$

$f'(4) = -32 + 16$

$f'(4) = -16 \text{ m/s}$

To get a numerical value for the speed we must specify a value for t, in this case $t = 4$.

Note:
-16 m/s is actually the **velocity** of the ball.
The speed of the ball is 16 m/s. Speed is a scalar quantity. It is the magnitude of the velocity.

Use your answer from **part (d)(i)** to find the value of t for which the ball was descending and travelling at a speed of 8 metres per second.

$f(t) = -4t^2 + 16t + 1$

$f'(t) = -8t + 16$

In (d)(i) we differentiated the function to create a formula for the speed of the ball. Let this formula for the speed (velocity) equal to -8 (the ball is descending so the rate of change of the height is negative) and solve for t.

$$-8t + 16 = -8$$

$$-8t = -8 - 16$$

$$-8t = -24$$

$$t = \frac{-24}{-8}$$

$$t = 3$$

The ball was **descending** and travelling at a speed of 8 metres per second after 3 seconds.