

# LCOL BASIC SKILLS PACK 2 

LEAVING CERT ORDINARY LEVEL

## LCOL Basic Skills: Pack 2 - Table of Contents

## Topic, Year and Level

1 Complex Numbers : 2006 Paper 1 - Q4 (c) (i)
2 Area, Perimeter \& Volume : 2010 NCAA Paper 2-Q1 (b)
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4 Trigonometry: 2009 Paper 2 - Q5 (c) (i)
5 Geometry: 2003 (JCHL) Paper 2 - Q4 (a)


Maths Points

## Division in Complex Numbers

To divide by a complex number we multiply above and below by the conjugate of the denominator．

To find the conjugate of a complex number we change the sign of the imaginary component．

$$
\begin{aligned}
\frac{\text { Complex Conjugate }}{1-4 i}=1+4 i & =\frac{3+12 i-2 i-8 i^{2}}{1+4 i-4 i-16 i^{2}} \\
& =\frac{3+10 i-8(-1)}{1-16(-1)} \\
& =\frac{3+10 i+8}{1+16} \\
& =\frac{11+10 i}{17} \\
& =\frac{11}{17}+\frac{10}{17} i
\end{aligned}
$$

$$
\begin{aligned}
& \frac{3-2 i}{1-4 i} \\
& =\frac{3-2 i}{1-4 i} \times \frac{1+4 i}{1+4 i} \\
& =\frac{3(1+4 i)-2 i(1+4 i)}{1(1+4 i)-4 i(1+4 i)} \\
& =\frac{3+12 i-2 i-8 i^{2}}{1+4 i-4 i-16 i^{2}} \\
& =\frac{3+10 i-8(-1)}{1-16(-1)} \\
& =\frac{3+10 i+8}{1+16} \\
& =\frac{11+10 i}{17} \\
& =\frac{11}{17}+\frac{10}{17} i
\end{aligned}
$$

Use the Trapezoidal Rule to determine which of the shapes A or B below has the greater area，and by how much．

The formula for the Trapezoidal Rule is on page 12 of the Maths Formulae Book．

## Trapezoidal Rule

$A=\frac{h}{2}\left(y_{1}+y_{n}+2\left(y_{2}+y_{3}+\cdots+y_{n-1}\right)\right)$
$A=\frac{h}{2}($ First + Last $+2($ Sum of the Rest $))$

$A=\frac{15}{2}(13+10+2(16+9+13+13+13))$

$$
\begin{aligned}
& h=\frac{90}{6} \\
& h=15
\end{aligned}
$$

$A=1132.5 \mathrm{~m}^{2}$
$A=\frac{h}{2}($ First + Last $+2($ Sum of the Rest $))$
$A=\frac{21}{2}(0+16+2(11+12+11))$
$A=882 \mathrm{~m}^{2}$

Thus，shape A has the greater area．
It is greater by $1132.5-882=250.5 \mathrm{~m}^{2}$


$$
\begin{aligned}
& h=\frac{84}{4} \\
& h=21
\end{aligned}
$$

Let $f(x)=x^{3}-6 x^{2}+9 x-3$ ．
Find $f^{\prime}(x)$ ，the derivative of $f(x)$ ．

## Differentiation－Power Rule

$f(x)=x^{n}$
$f^{\prime}(x)=n x^{n-1}$
For each term bring the power to the front （multiply）and reduce the power by 1.

The derivative of any constant（a term independent of $x$ ）is 0 ．

## Lagrange＇s Notation

The notation in this question is called prime notation（or Lagrange＇s notation）． The＇prime＇is a tick mark places after the function symbol，$f$ ．
$f^{\prime}(x)$ reads＇f prime of $x$＇．
Joseph－Louis Lagrange（1736－1813）was an Italian mathematician．

$$
\begin{aligned}
& f(x)=x^{3}-6 x^{2}+9 x-3 \\
& f^{\prime}(x)=3 x^{2}-12 x+9
\end{aligned}
$$

## Leibniz＇s Notation

$y=x^{3}-6 x^{2}+9 x-3$
$\frac{d y}{d x}=3 x^{2}-12 x+9$


Leibniz and Isaac Newton worked independently of each other in the creation of＇calculus＇．

A harbour is 6 km due East of a lighthouse.
A boat is 4 km from the lighthouse.
The bearing of the boat from the lighthouse is $\mathrm{N} 40^{\circ} \mathrm{W}$.

The Cosine Rule is on page 16 of the Maths Formulae Book.

How far is the boat from the harbour?
Give your answer correct to one decimal place.
$40+90=130^{\circ}$

Let $x$ be the distance from the Boat to the Harbour.

## Cosine Rule

$$
a^{2}=b^{2}+c^{2}-2 b c \cos A
$$

$$
a^{2}=b^{2}+c^{2}-2 b c \cos A
$$

$$
x^{2}=4^{2}+6^{2}-2(4)(6) \cos 130^{\circ}
$$

$$
x^{2}=82.85
$$

$$
x=\sqrt{82.85}
$$

$$
x=9.1 \mathrm{~km}
$$



In the parallelogram $a b c d,|\angle a b c|=53^{\circ}$ and $|b c|=12 \mathrm{~cm}$.
(i) Find $|\angle b c d|$.
(ii) Find the perpendicular height, $h$, given that the area of $a b c d$ is $90 \mathrm{~cm}^{2}$.

## Theorem 9

In a parallelogram, opposite sides are equal, and opposite angles are equal.
$|\angle b c d|=\frac{1}{2}(360-2(53))$

$|\angle b c d|=\frac{1}{2}(360-106)$
$|\angle b c d|=\frac{1}{2}(254)$
$|\angle b c d|=127^{\circ}$

| Area of a Parallelogram <br> $A=$ base $\times$ perpendicular height |  |
| ---: | :--- |
| The formula for the Area of a <br> Parallelogram is on page 8 of | $h=\frac{90}{12}$ |

Let the formula for the area of a parallelogram equal 90 and solve for the height $h$.


