

# LCOL BASIC SKILLS PACK 4 

LEAVING CERT ORDINARY LEVEL

## LCOL Basic Skills: Pack 4 - Table of Contents

## Topic, Year and Level

1 Algebra: 2011 Paper 1 - Q3 (c)
$2>$ Differentiation: 2003 Paper 1-Q8 (c)
Sequences and Series (Patterns) : 2011 Paper 1-Q5 (b)
4 Coordinate Geometry: 2010 Paper 2 - Q3 (a)
5 Statistics: 2015 LCOL Strand 1 Supplementary Paper - Q1 (a)


Maths Points

Solve for $x$

$$
\frac{x-1}{x}+\frac{x}{x+1}=\frac{1}{2} \quad x \neq 0, x \neq-1 .
$$

$$
\begin{aligned}
& \frac{x-1}{x}+\frac{x}{x+1}=\frac{1}{2} \\
& \frac{2(x+1)(x-1)+2(x)(x)=1(x)(x+1)}{\underline{2 x(x+1)}} \\
& 2\left(x^{2}-1\right)+2 x^{2}=x^{2}+x \\
& 2 x^{2}-2+2 x^{2}=x^{2}+x \\
& 3 x^{2}-x-2=0 \\
& (3 x+2)(x-1)=0 \\
& 3 x+2=0
\end{aligned} \quad x-1=0, \quad x=1 .
$$

```
Note:
Numerator - top of fraction
Denominator - bottom of fraction
```

The common denominator is the product of the denominators of each term. Multiply each numerator by the denominators of the other terms. We can remove the denominator because there is an equals on top.

Solve the quadratic equation by factorising or the $-b$ formula.

Let $f(x)=x^{3}+2 x^{2}-1$.
$L$ is the tangent to the curve $y=f(x)$ at $x=\frac{-2}{3}$.
Find the slope of $L$

$$
\begin{aligned}
& f(x)=x^{3}+2 x^{2}-1 \\
& f^{\prime}(x)=3 x^{2}+4 x \\
& f^{\prime}\left(\frac{-2}{3}\right) \\
& =3\left(\frac{-2}{3}\right)^{2}+4\left(\frac{-2}{3}\right) \\
& =\frac{4}{3}-\frac{8}{3} \\
& =-\frac{4}{3}
\end{aligned}
$$

When you differentiate a function, you get the slope of that function.
To get a numerical value for the slope we must specify a value for $x$, in this case $x=\frac{-2}{3}$.

The slope of the tangent to the curve at $x=\frac{-2}{3}$ is $-\frac{4}{3}$.

## Sum of an Arithmetic Series

$S_{n}=\frac{n}{2}(2 a+(n-1) d)$

$$
\begin{aligned}
& a=7 \\
& d=-3
\end{aligned}
$$

$$
\begin{aligned}
& S_{n}=\frac{n}{2}(2 a+(n-1) d) \\
& S_{15}=\frac{15}{2}(2(7)+(15-1)(-3)) \\
& S_{15}=7.5(14+14(-3)) \\
& S_{15}=7.5(14-42) \\
& S_{15}=7.5(-28) \\
& S_{15}=-210
\end{aligned}
$$

The sum of the first fifteen term of the series is -210 .

A circle with centre $(0,0)$ passes through the point $(5,-12)$.
(i) Find the radius of the circle.
(ii) Write down the equation of the circle.

## Distance Formula

$d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$

| $(0,0)$ | $(5,-12)$ |
| :--- | :--- |
| $\left(x_{1}, y_{1}\right)$ | $\left(x_{2}, y_{2}\right)$ |

$r=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
$r=\sqrt{(5-0)^{2}+(-12-0)^{2}}$
$r=\sqrt{(5)^{2}+(-12)^{2}}$
$r=\sqrt{25+144}$
$r=\sqrt{169}$
$r=13$

The Equation of a Circle
formula is on page 19 of
the Maths Formulae Book.

## (ii)

$$
\begin{aligned}
& \begin{array}{l}
\text { Standard Equation of a Circle } \\
(x-h)^{2}+(y-k)^{2}=r^{2}
\end{array} \\
& \text { Centre, }(h, k) \quad \begin{array}{l}
(h, k)=(0,0) \\
\text { Radius, } r \\
r=13
\end{array} \\
& (x-h)^{2}+(y-k)^{2}=r^{2} \\
& (x-0)^{2}+(y-0)^{2}=13^{2} \\
& x^{2}+y^{2}=169
\end{aligned}
$$

## Alternately

Equation of a Circle with centre ( 0,0 ) and radius, $r$.
$x^{2}+y^{2}=r^{2}$

$$
\begin{aligned}
& x^{2}+y^{2}=r^{2} \\
& x^{2}+y^{2}=13^{2} \\
& x^{2}+y^{2}=169
\end{aligned}
$$

A survey is being conducted of voters' opinions on several different issues.
What is the overall margin of error of the survey, at 95\% confidence, if it is based on a simple random sample of 1111 voters?

Margin of Error
$E=\frac{1}{\sqrt{n}}$
where $n$ is the sample size.

$$
n=1111
$$

$$
\begin{aligned}
& E=\frac{1}{\sqrt{n}} \\
& E=\frac{1}{\sqrt{1111}} \\
& E \approx 0.03 \\
& E \approx 3 \%
\end{aligned}
$$

A political party had claimed that it has the support of $24 \%$ of the electorate. Of the voters in the sample above, 243 stated that they support the party.
Is this sufficient evidence to reject the party's claim, at the $5 \%$ level of significance?

## Null Hypothesis:

$H_{0}: p=0.24$, the percentage of people who support the political party is $24 \%$.
Alternate Hypothesis:
$H_{1}: p \neq 0.24$, the percentage of people who support the political party is NOT $24 \%$.
Sample Proportion
$\hat{p}=\frac{243}{1111}$
$\hat{p}=0.2187$

## Margin of Error

$E=0.03$

Confidence Interval
$\hat{p} \pm \frac{1}{\sqrt{n}}$
$\hat{p}-\frac{1}{\sqrt{n}}<p<\hat{p}+\frac{1}{\sqrt{n}}$
$0.2187-0.03<p<0.2187+0.03$
$0.1887<p<0.2487$

We are 95\% confident that between $18.87 \%$ and $24.87 \%$ of the electorate support the political party.

As 24\% lies within this confidence interval there is insufficient evident to reject the part's claim.


