Maths Points

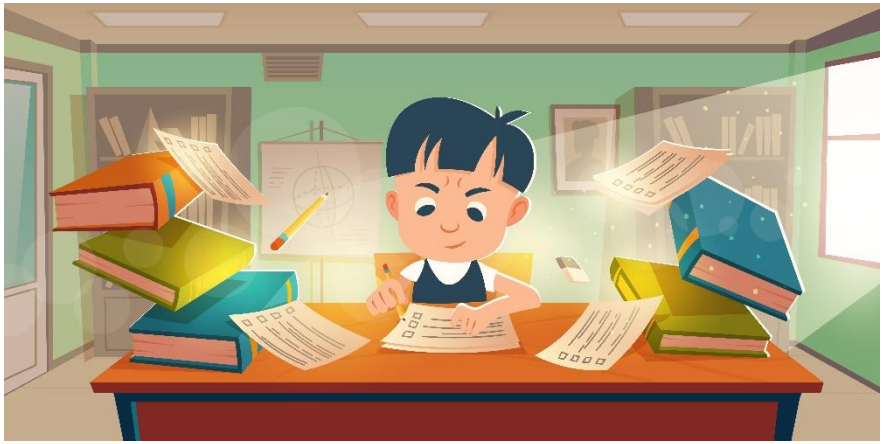
Junior and Leaving Cert

LCOL BASIC SKILLS

PACK 7

Leaving Cert Ordinary Level

LCOL Basic Skills: Pack 7 – Table of Contents



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Solve for x and y

$x - y + 5 = 0$ ← Linear

$x^2 + y^2 = 17$ ← Non-Linear

Take the **linear** expression and express x in terms of y.

$x - y + 5 = 0$

$x = y - 5$

Substitute this value of x into the **non-linear** equation.

$x^2 + y^2 = 17$

$(y - 5)^2 + y^2 = 17$

$(y - 5)(y - 5) + y^2 = 17$

$y(y - 5) - 5(y - 5) + y^2 = 17$

$y^2 - 5y - 5y + 25 + y^2 = 17$

$2y^2 - 10y + 8 = 0$

$y^2 - 5y + 4 = 0$ ← Factorise to solve the quadratic.

$(y - 1)(y - 4) = 0$

$y - 1 = 0$ | $y - 4 = 0$

$y = 1$ | $y = 4$

We can now use our y values to solve for x by subbing them back into the rearranged linear equation.

$x = y - 5$

$x = 1 - 5$ ← $y = 1$

$x = -4$

$(-4, 1)$

$x = y - 5$

$x = 4 - 5$ ← $y = 4$

$x = -1$

$(-1, 4)$

Given that $i^2 = -1$, simplify

$$2(3 - 5i) + 7i(2 + 3i)$$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$.

Expand brackets by multiplying

$2(3 - 5i) + 7i(2 + 3i)$

$= 6 - 10i + 14i + 21i^2$

$= 6 - 10i + 14i + 21(-1)$ ← $i^2 = -1$

$= 6 + 4i - 21$

$= -15 + 4i$

The volume of a hemisphere is 486π cm^3.

Find the radius of the hemisphere.

Volume of a Sphere

$V = \frac{4}{3}\pi r^3$

Volume of a Hemisphere

$V = \frac{2}{3}\pi r^3$

We are given the volume of the hemisphere so let this equal to the formula and solve for r.

$$\frac{2}{3}\pi r^3 = 486\pi$$

$$\frac{2}{3}r^3 = 486$$

$$r^3 = \frac{486}{\frac{2}{3}}$$

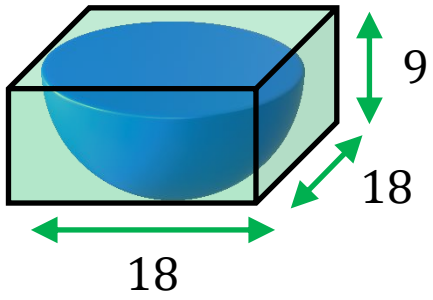
$$r^3 = 729$$

$$r = 9 \text{ cm}$$

The formulae for the **Volume of a Sphere** is on **page 10** of the Maths Formulae Book.

(ii)

Find the volume of the smallest rectangular box that the hemisphere will fit into.



Radius = 9

Diameter = 18

Volume of a Cuboid

V = Length × Breadth × Height

$$V = l \times b \times h$$

$$V = 18 \times 18 \times 9$$

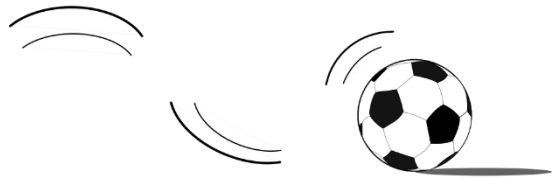
$$V = 2916 \text{ cm}^3$$

A ball is rolled in a straight line along a surface.
The distance, s metres, the ball travels is given by

$$s = 18t - 2t^2$$

where t is the time in seconds from the instant the ball begins to move.

Find the speed of the ball after 3 seconds.



Differentiate a distance formula to get a formula for speed.

$s = 18t - 2t^2$ ← Distance

$\frac{ds}{dt} = 18 - 4t$ ← Speed

When you differentiate a function, you get the **rate of change** of that function.

Speed is the change in distance with respect to time!

To get a numerical value for the speed we must specify a value for t, in this case $t = 3$.

Calculate the speed when the time is 3 seconds, $t = 3$.

$\frac{ds}{dt} = 18 - 4t$

$= 18 - 4(3)$ ← $t = 3$

$= 18 - 12$

$= 6$ m/s

Continued

How far is the ball from the starting point when it stops moving?

The ball will stop when its speed is 0. Calculate the time taken for this to happen and then sub this time into the distance formula.

$s = 18t - 2t^2$

$\frac{ds}{dt} = 18 - 4t$

$18 - 4t = 0$

$-4t = -18$

$t = \frac{-18}{-4}$

$t = 4.5$ seconds

$s = 18t - 2t^2$

$s = 18(4.5) - 2(4.5)^2$ ← $t = 4.5$

$s = 81 - 40.5$

$s = 40.5$ m

The ball will have stopped after 4.5 seconds and will have travelled 40.5 m in that time.

Continued

Show that the speed of the ball decreases at a constant rate while it is moving.

If we are given a distance formula, represented by s, we can find formula for speed and acceleration by differentiating.

Acceleration is the change in speed over time.

$s = 18t - 2t^2$

$\frac{ds}{dt} = 18 - 4t$

$s = 18t - 2t^2$ ← Distance

$\frac{ds}{dt} = 18 - 4t$ ← Speed

$\frac{d^2s}{dt^2} = -4$ ← Acceleration

The acceleration is a constant -4 m/s^2 therefore the speed of the ball decreases at a constant rate while it is moving.

The terms in an arithmetic sequence are given by the formula

$$T_n = 38 - 4n, \qquad \text{for } n = 1, 2, 3, 4, \ldots$$

Write out the first three terms in the sequence.

$T_n = 38 - 4n$

$T_n = 38 - 4n$

$T_1 = 38 - 4(1) = 34$

$T_2 = 38 - 4(2) = 30$

$T_3 = 38 - 4(3) = 26$

$34, 30, 26 \ldots$

(b)

What is the first negative term of the sequence.

Looking for the value of n that makes the term negative (< 0).

$38 - 4n < 0$

$38 < 4n$

$9.5 < n$

10th term is the first negative term.

Sub $n = 10$ into the formula to find the 10^{th} term (the first negative term).

$T_n = 38 - 4n$

$T_{10} = 38 - 4(10)$

$T_{10} = 38 - 40$

$T_{10} = -2$

$34, 30, 26, 22, 18, 14, 10, 6, 2, -2 \ldots$

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