Maths Points

Junior and Leaving Cert

PAPER 1: FORMAL PROOFS AND CONSTRUCTIONS

Leaving Cert Higher level

Paper 1 Formal Proofs and Constructions

Proofs

- Proof By Contradiction – $\sqrt{2}$ is not Rational
- Sum of Geometric Series by Induction
- Sum to Infinity of Geometric Series (Limits)
- Derive Amortisation Formula
- De Moivre's Theorem by Induction

Constructions

- Construction of $\sqrt{2}$
- Construction of $\sqrt{3}$

Learn Also

- De Moivre to Prove Trigonometric Identity I
- De Moivre to Prove Trigonometric Identity II
- Differentiate by 1st Principles

Maths Points

Junior and Leaving Cert

Prove, by induction, the formula for the sum of the first n terms of a geometric series. That is, prove that, for $r \neq 1$:

$$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

Step 1: Show true for $n = 1$

$$a = \frac{a(1-r^1)}{1-r}$$

$$a = a$$

which is true

Step 2: Assume true for $n = k$

$$a + ar + ar^2 + \cdots + ar^{k-1} = \frac{a(1-r^k)}{1-r}$$

Step 3: Prove true for $n = k + 1$

$$a + ar + ar^2 + \cdots + ar^{k-1} + ar^k = \frac{a(1-r^{k+1})}{1-r}$$

Proof

Begin with the assumption

$$a + ar + ar^2 + \cdots + ar^{k-1} = \frac{a(1-r^k)}{1-r}$$

$$a + ar + ar^2 + \cdots + ar^{k-1} + ar^k = \frac{a(1-r^k)}{1-r} + ar^k$$

$$= \frac{a(1-r^k) + ar^k(1-r)}{1-r}$$

$$= \frac{a\left(1 - r^k + r^k(1-r)\right)}{1-r}$$

$$= \frac{a(1 - r^k + r^k - r^{k+1})}{1-r}$$

$$= \frac{a(1-r^{k+1})}{1-r}$$

Add ar^k to both sides of the assumption and then work with RHS only.

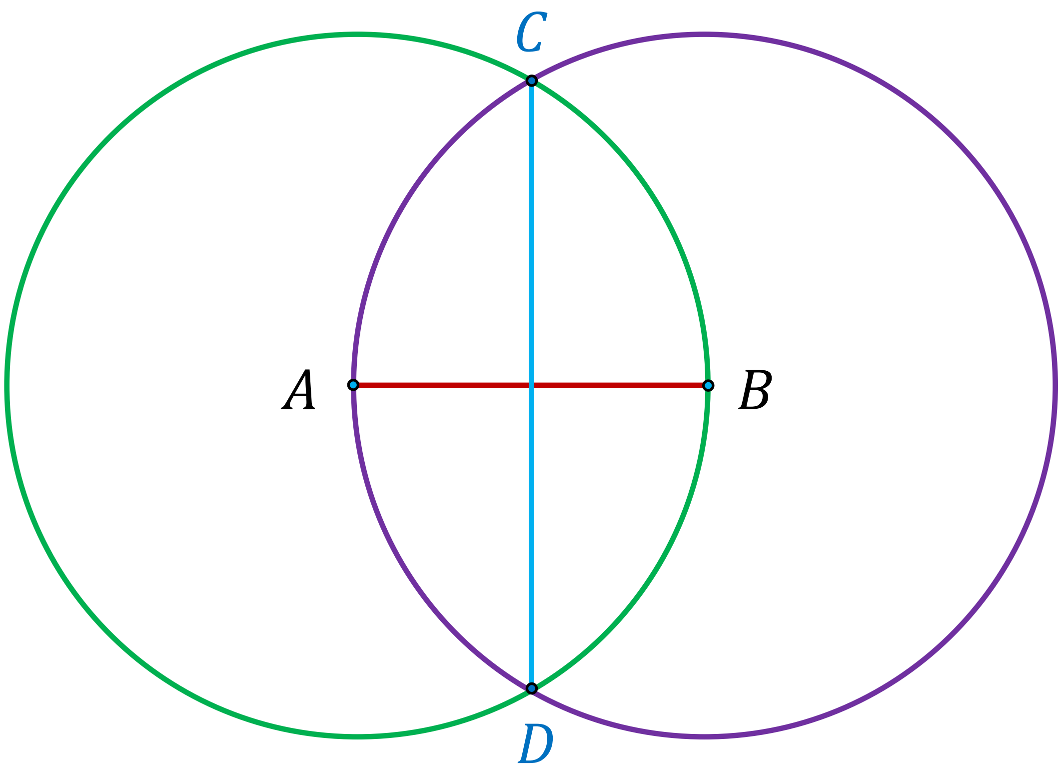
$r^k . r = r^{k+1}$

The proposition is true for $n = 1$. If the proposition is true for $n = k$, then it will be true for $n = k + 1$. Therefore, by induction it is true for all $n \in N$.

Steps

1. Let the line segment AB be of length 1 unit.
2. Construct a circle with centre A and radius length |AB|.
3. Construct a circle with centre B and radius length [AB].
4. Mark the intersection of the two circles as C and D.
5. Draw the line segment [CD].

$$|CD| = \sqrt{3}$$



Maths Points

Junior and Leaving Cert