

► 2024 LCHL Paper 1 – Question 10

A company makes windscreen wipers.

In this question, the rectangle $PQRS$ has a width of 180 cm and a height of 100 cm.

M is the midpoint of $[PQ]$, N is the midpoint of $[SR]$, and $O \in NM$. All lengths are given in cm.

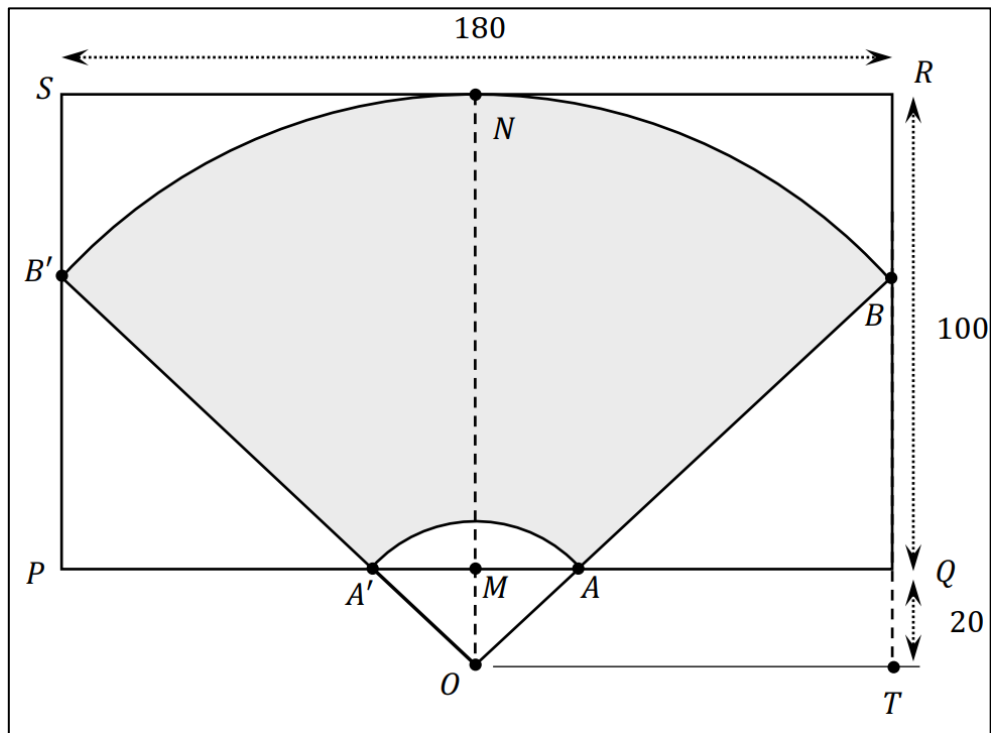
(a) In the diagram below, the line segment $[AB]$ shows a type of wiper blade.

$[AB]$ rotates around the point O , where $O \in AB$, until it reaches the position $[A'B']$.

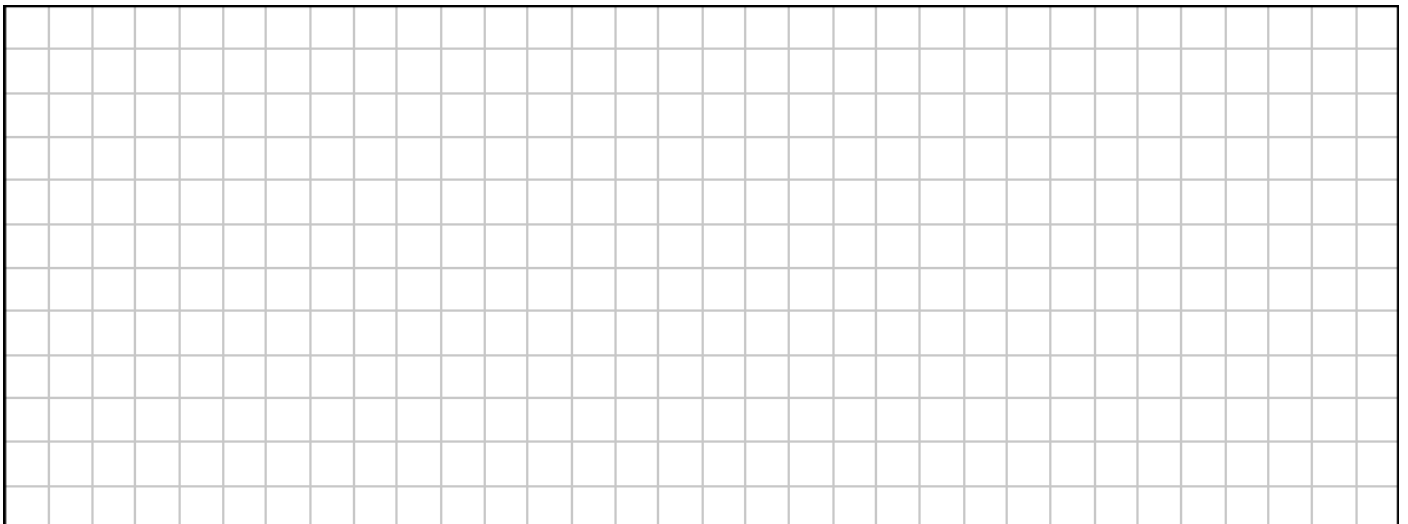
The region that it cleans is $ABB'A'$, which is the sector OBB' with the sector OAA' removed.

A, A', B , and B' lie on the rectangle $PQRS$, and N lies on the arc from B to B' .

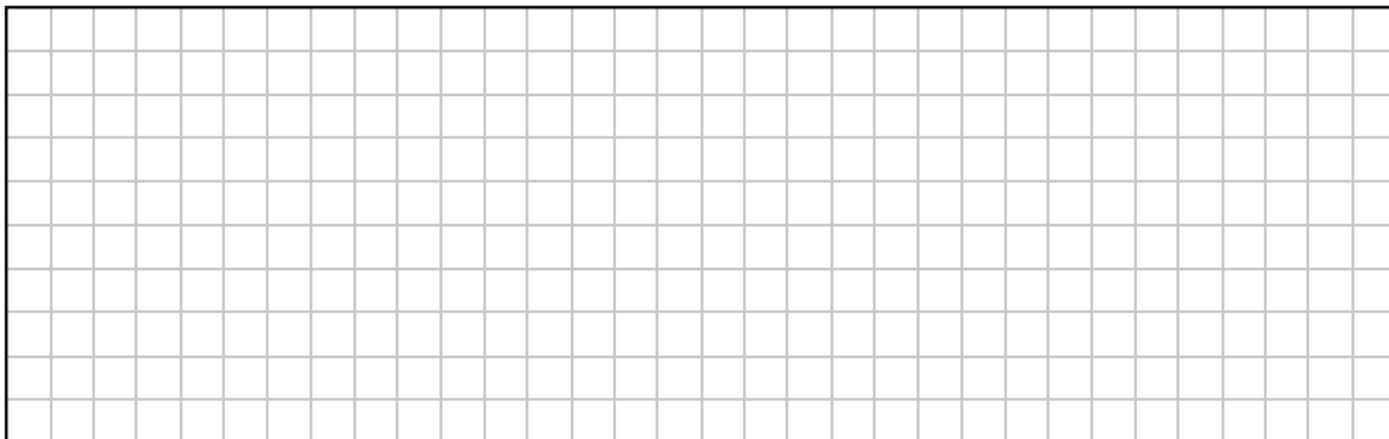
The line segment $[RQ]$ is extended 20 cm to T , as shown. $\angle OTP$ is a right angle.



(i) Show that $|OB| = 120$ cm.



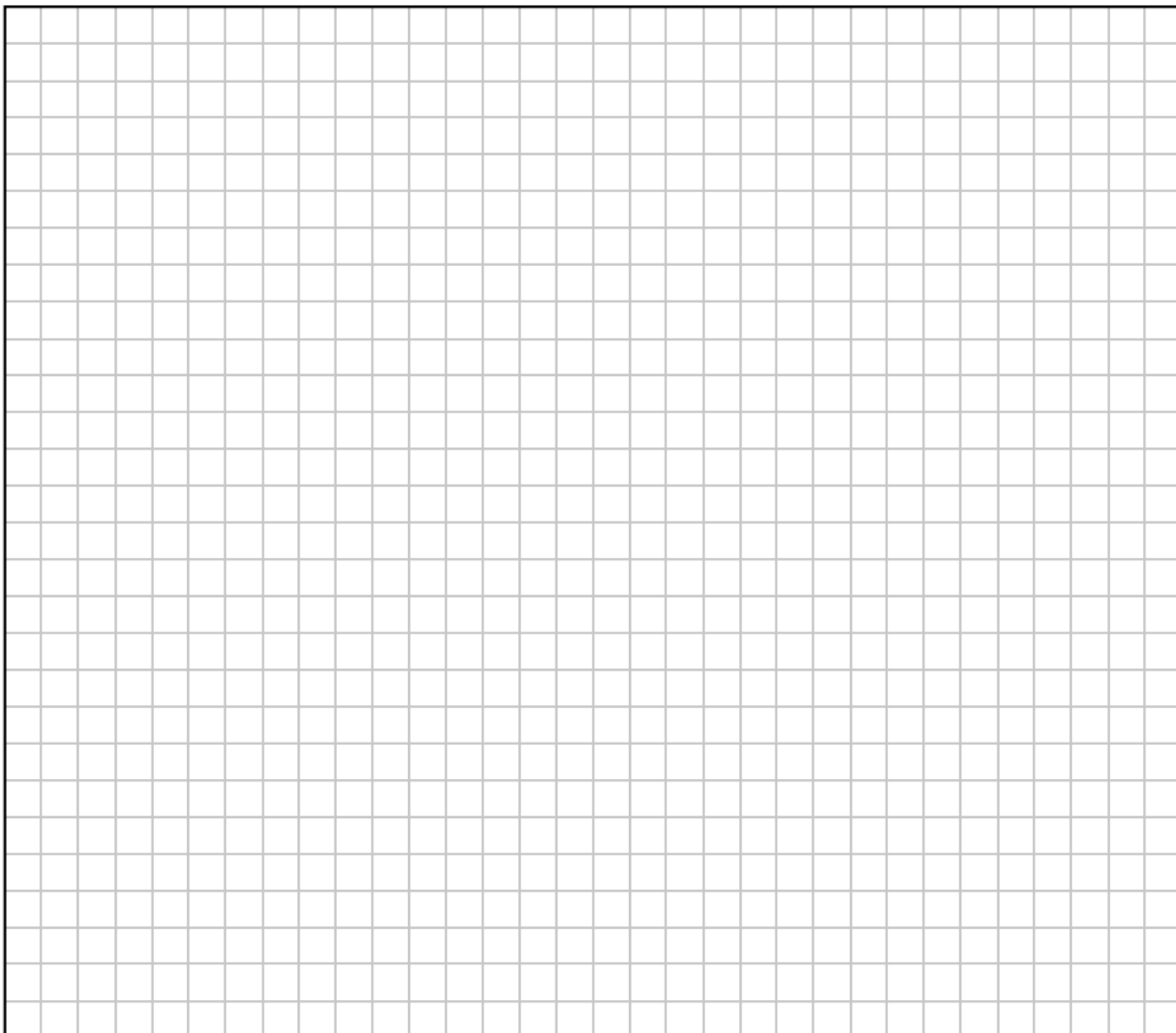
(ii) Hence, show that $|\angle BOT| = 41.4^\circ$, correct to 1 decimal place.



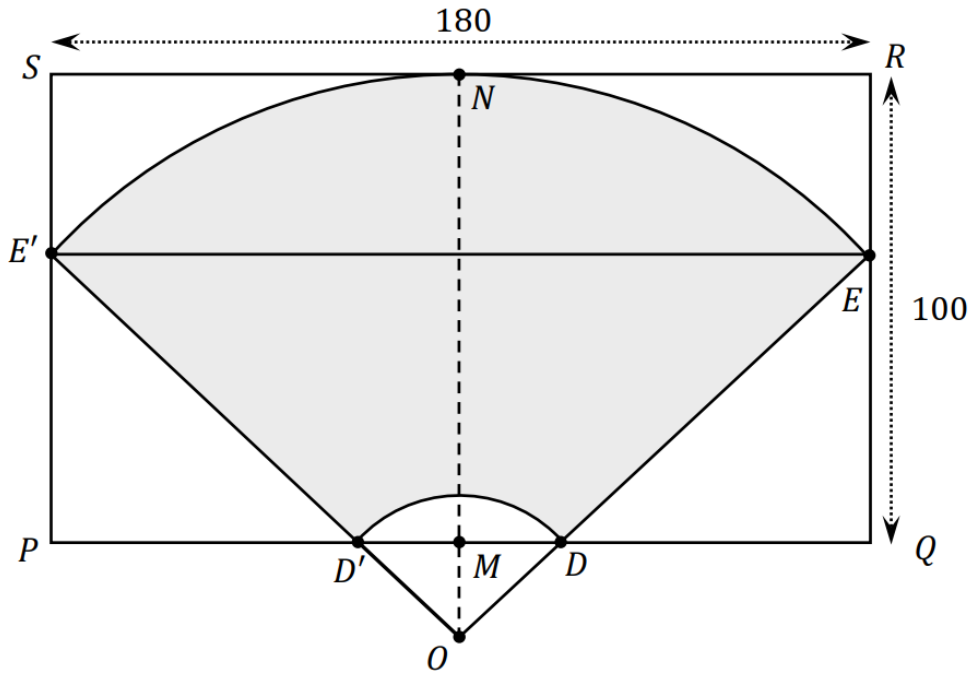
(iii) Hence, work out the area of $ABB'A'$.

Remember that $ABB'A'$ is the sector OBB' with the sector OAA' removed.

Give your answer correct to the nearest cm^2 .



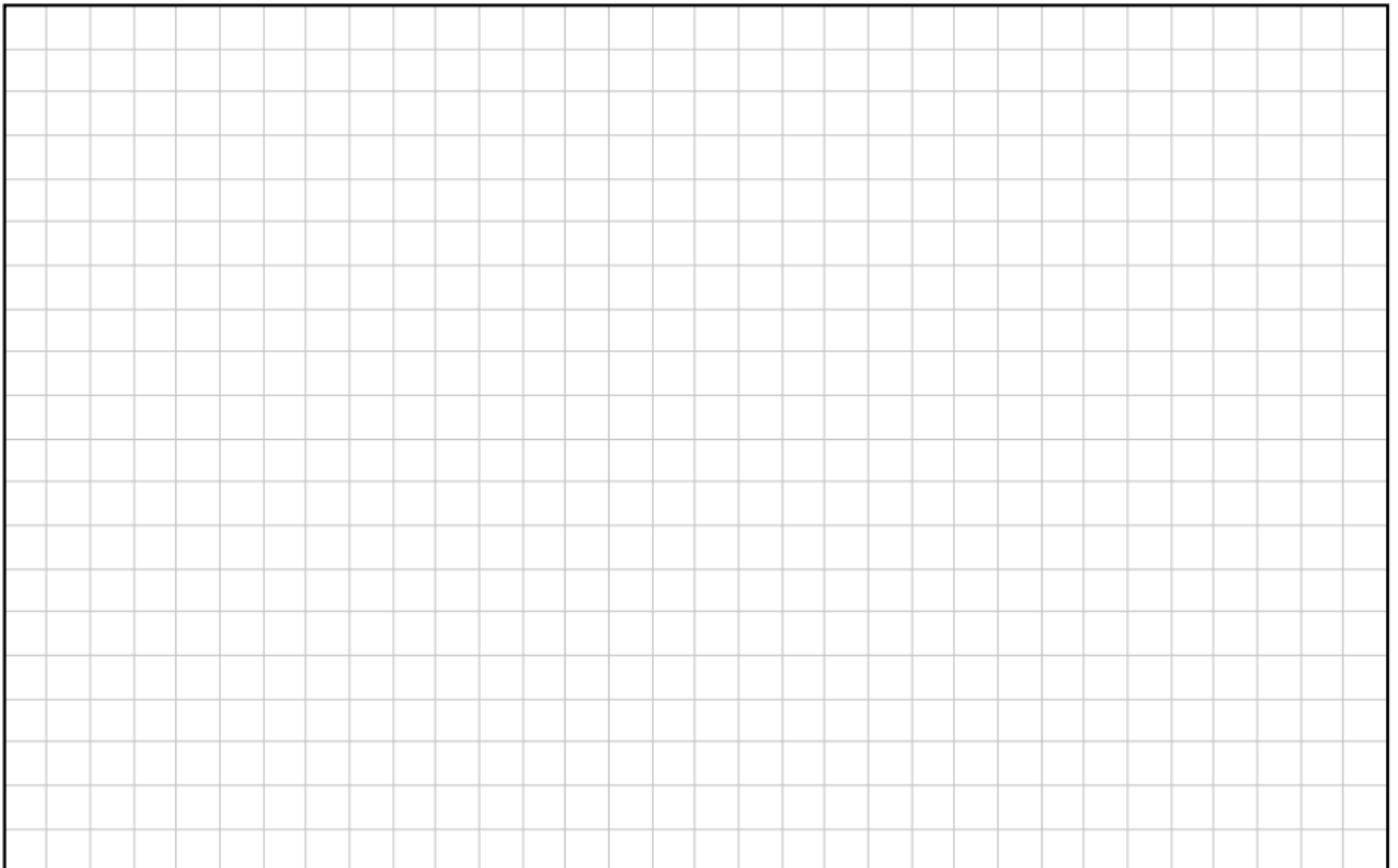
(b) Moving the point O along the line NM changes the size of the wiper blade and the region that it cleans. In the diagram below, $[DE]$ rotates about O , where $O \in DE$, until it reaches $[D'E']$. As in part (a), E and E' lie on the rectangle $PQRS$.



Here, $|\angle E'OE| = 105^\circ$.

By setting $|OE| = x$, use the **triangle** OEE' and the **Cosine Rule** to find the value of $|OE|$.

Give your answer in cm, correct to 1 decimal place.



(c) Mattie is driving home. On the way, she passes five traffic lights. Each traffic light is either red (R), green (G), or orange (O) when she arrives at it. One day, she notes the pattern made by the colour of each traffic light when she arrives at it. For example, this pattern could be **R R O G R**.

(i) How many different patterns could the five traffic lights make?

(ii) How many different patterns could the five traffic lights make, if the first light is red **and** the fifth light is **not** red?

(iii) How many different patterns could the five traffic lights make if no two consecutive lights are the same colour?