2024 LCHL Paper 2 Q8

This section is to be completed having corrected the question. **Enter your score in the box above ↑.**

Thoughts (Easy/ Hard, State the Topic, Links to other Topics, Similar Questions?)

What formulae or rules did I need? Is there anything I should learn off?

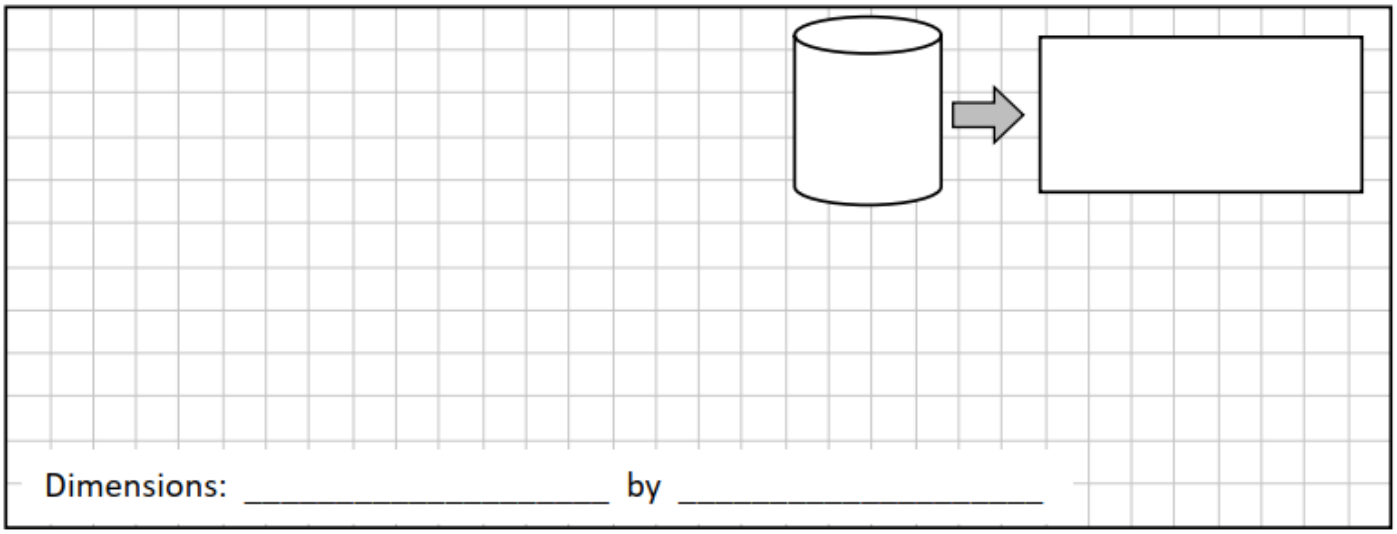
Is there anything I still don't understand? What do I need to revise? Should I return to this question?

▶ 2024 LCHL Paper 1 – Question 8

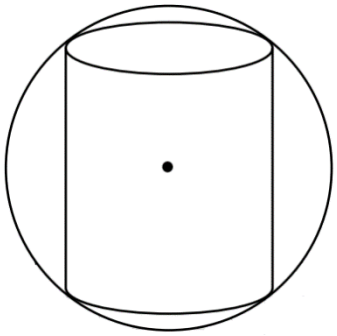
Tommy makes ornaments from metal and glass.

(a) He makes an open metal cylinder with a height of 15 cm and a radius of 5 cm.
The **net** of this cylinder is a rectangle.

Find the dimensions of this rectangle.
Give your answers in cm, correct to 1 decimal place where appropriate.



(b) Tommy makes another cylinder with a height of 22 cm and a diameter of 12 cm.

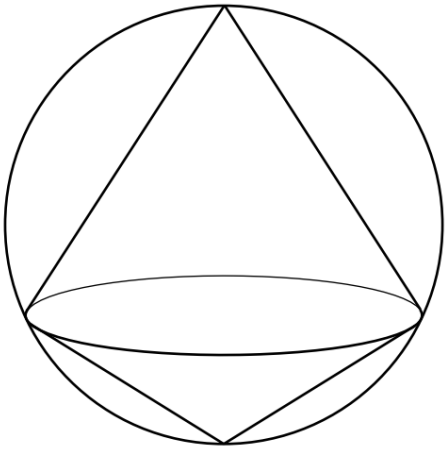


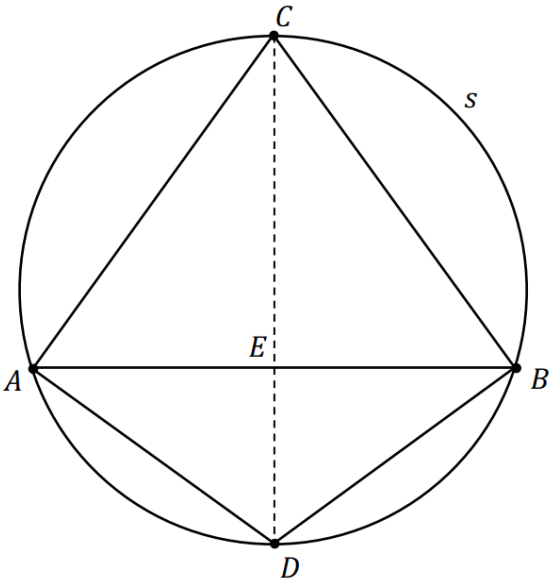
This cylinder fits exactly inside a glass sphere.
The top and bottom edges of the cylinder touch the sphere.

Find the **volume** of the **sphere**, in cm^3, correct to 1 decimal place. Use the Theorem of Pythagoras in your solution.

(c) Another ornament is made of two cones inscribed in a sphere.
The top cone is upright; the bottom cone is inverted. The cones have the same base.

A vertical cross-section of the ornament, taken through the centre of the sphere, shows the cones as two triangles, ABC and ADB, with a common side $[AB]$. ABC is the top cone.
The points A, B, C, and D all lie on the circle s, which represents the cross-section of the sphere.
The lines AB and CD intersect at the point E.





(i) The diagram is symmetrical about the line DC. State why $|\angle CBD| = 90°$.

(ii) Hence, or otherwise, **prove** that the triangles BCE and DBE are **similar**.
Give a reason for each statement that you make, where appropriate.

(iii) The top cone has a radius of r and a height of h; that is, $|EB| = r$ and $|EC| = h$.
The sphere, represented by s, has a radius of 10 cm.

Use the similar triangles BCE and DBE to show that:

$$r^2 = 20h - h^2$$

(iv) Hence, write the volume of the top cone in terms of h and π, **and** find the value of h that gives the **maximum volume** for the top cone.

$h =$ _____