

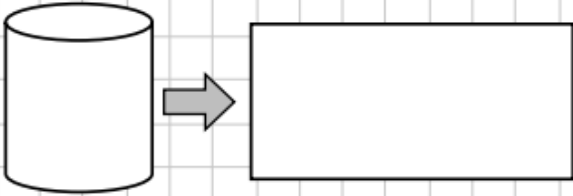
► 2024 LCHL Paper 1 – Question 8

Tommy makes ornaments from metal and glass.

- (a) He makes an open metal cylinder with a height of 15 cm and a radius of 5 cm.
The **net** of this cylinder is a rectangle.

Find the dimensions of this rectangle.

Give your answers in cm, correct to 1 decimal place where appropriate.



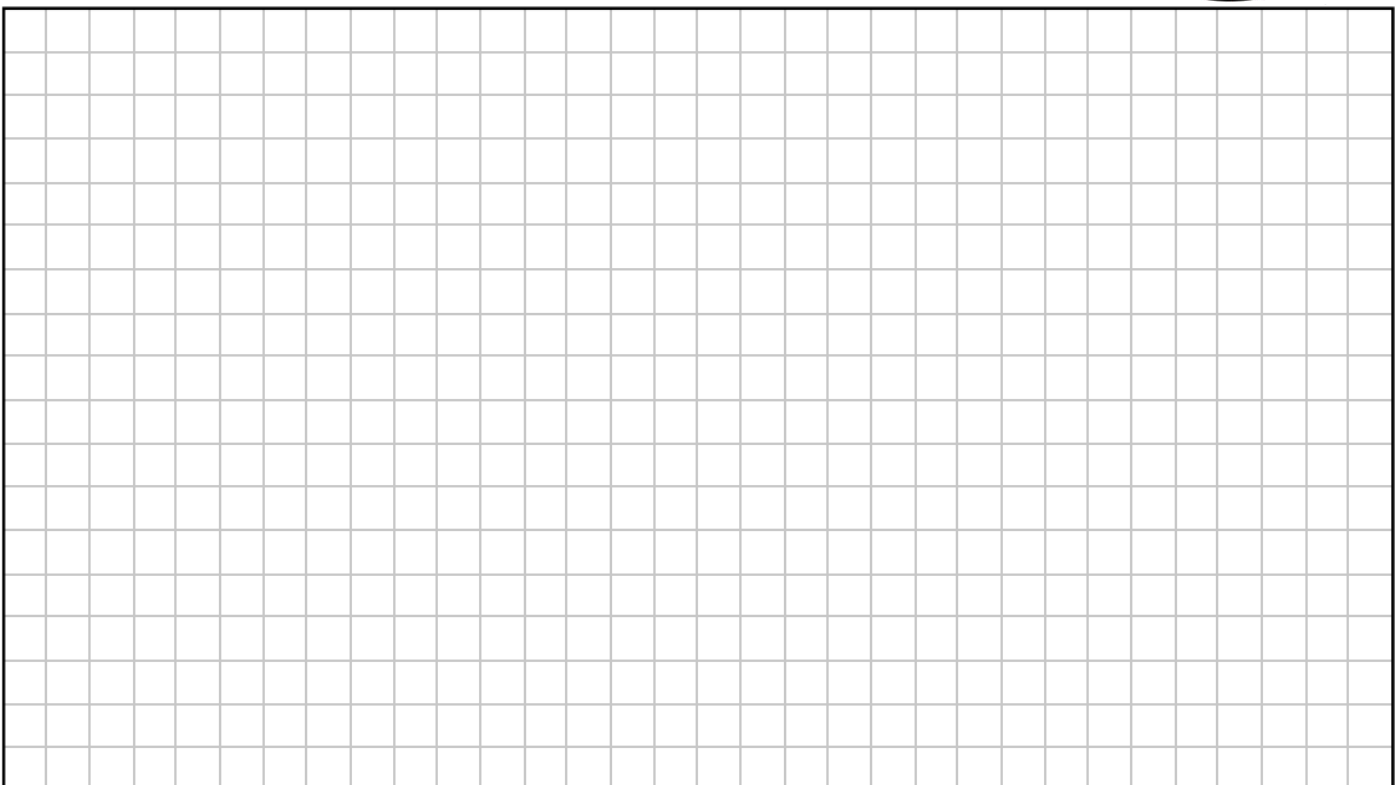
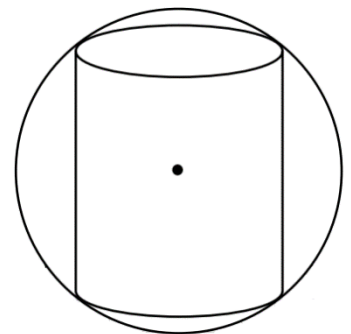
Dimensions: _____ by _____

- (b) Tommy makes another cylinder with a height of 22 cm and a diameter of 12 cm.

This cylinder fits exactly inside a glass sphere.

The top and bottom edges of the cylinder touch the sphere.

Find the **volume** of the **sphere**, in cm^3 , correct to 1 decimal place. Use the Theorem of Pythagoras in your solution.



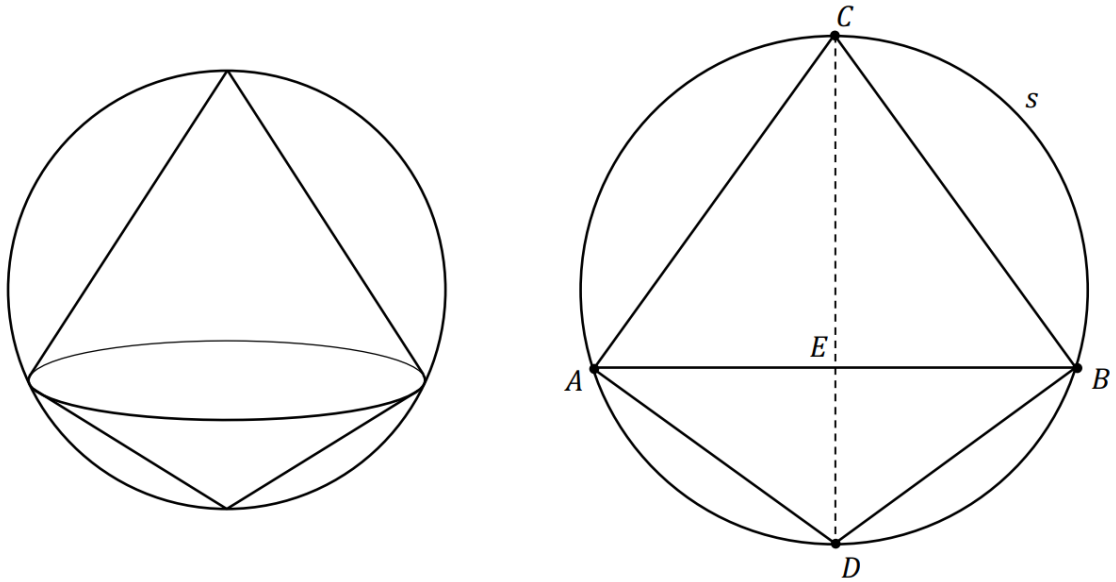
(c) Another ornament is made of two cones inscribed in a sphere.

The top cone is upright; the bottom cone is inverted. The cones have the same base.

A vertical cross-section of the ornament, taken through the centre of the sphere, shows the cones as two triangles, ABC and ADB , with a common side $[AB]$. ABC is the top cone.

The points A , B , C , and D all lie on the circle s , which represents the cross-section of the sphere.

The lines AB and CD intersect at the point E .



(i) The diagram is symmetrical about the line DC . State why $|\angle CBD| = 90^\circ$.

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(ii) Hence, or otherwise, **prove** that the triangles BCE and DBE are **similar**.

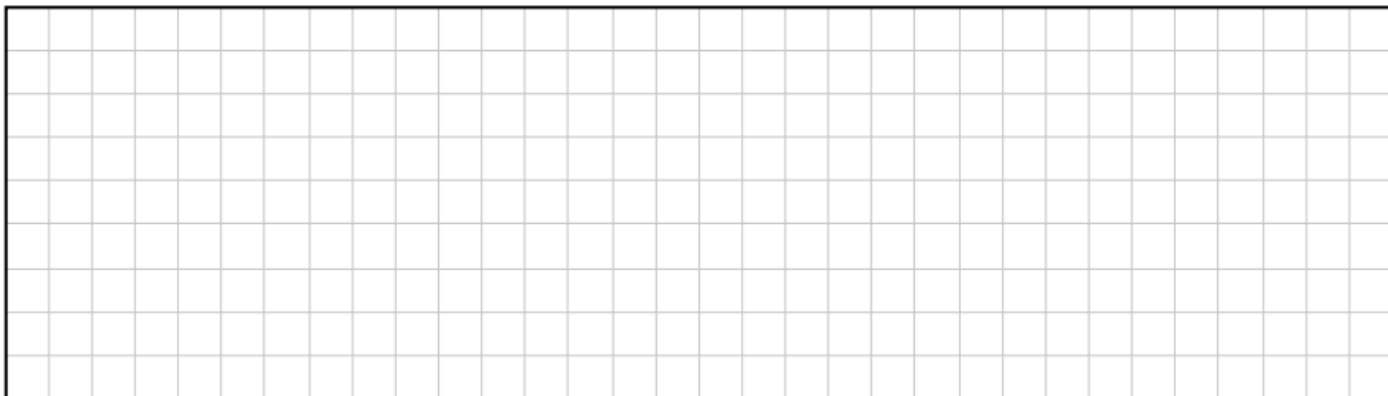
Give a reason for each statement that you make, where appropriate.

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- (iii) The top cone has a radius of r and a height of h ; that is, $|EB| = r$ and $|EC| = h$.
The sphere, represented by s , has a radius of 10 cm.

Use the similar triangles BCE and DBE to show that:

$$r^2 = 20h - h^2$$



- (iv) Hence, write the volume of the top cone in terms of h and π , **and** find the value of h that gives the **maximum volume** for the top cone.

