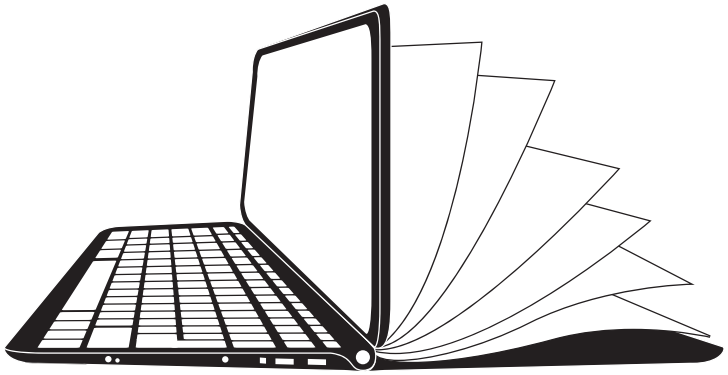


Maths Points

Junior and Leaving Cert

TRIGONOMETRY

Junior Cert Higher level

Trigonometry – Junior Cert Higher Level Revision Sheet

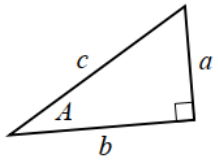
MATHSPOINTS.IE

Trigonometry is the study of triangles.

Solving Triangles

To find missing sides and angles in right angled triangles we can use **Pythagoras Theorem** or the **Trigonometric Ratios**. These are in the Formulae and Tables Booklet.

Pythagoras Theorem



$$c^2 = a^2 + b^2$$

where c is the **hypotenuse** of the triangle and a and b are the other two sides.

Trigonometric Ratios

$$\tan A = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin A = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Remember

Tom's $\frac{\text{Old}}{\text{Aunt}}$

Sat $\frac{\text{On}}{\text{Her}}$

Coat $\frac{\text{And}}{\text{Hat}}$

Rearranging Pythagoras Theorem

When looking for sides other than the hypotenuse it can be useful to rearrange the formula as this leaves the unknown on the left-hand side of the equation (it's optional).

$$a^2 + b^2 = c^2$$

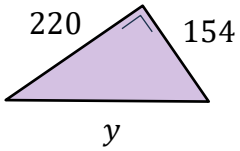
Using Ratios to Find Missing Sides & Angles

1. Label sides.
2. Pick a ratio
3. Form an equation.
4. Solve

When solving for an **angle** make sure to press SHIFT on calculator!

Using Pythagoras to Find Hypotenuse

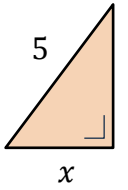
$$c^2 = a^2 + b^2$$
$$y^2 = 220^2 + 154^2$$
$$y^2 = 48400 + 23716$$
$$y^2 = 72116$$
$$y = \sqrt{72116}$$
$$y = 268.54$$



Using Pythagoras to Find Missing Side

$$a^2 + b^2 = c^2$$
$$4^2 + x^2 = 5^2$$
$$16 + x^2 = 25$$
$$x^2 = 9$$
$$x = 3$$

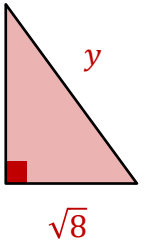
Formula in this form.



Pythagoras with Surds

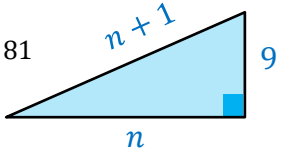
$$c^2 = a^2 + b^2$$
$$y^2 = \sqrt{8}^2 + 3^2$$
$$y^2 = 8 + 9$$
$$y^2 = 17$$
$$y = \sqrt{17} \text{ units}$$

Note: $\sqrt{x}^2 = x$



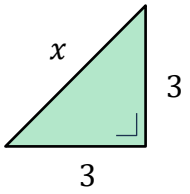
Pythagoras with Two Unknowns

$$c^2 = a^2 + b^2$$
$$(n+1)^2 = n^2 + 9^2$$
$$n^2 + 2n + 1 = n^2 + 81$$
$$2n + 1 = 81$$
$$2n = 80$$
$$n = 40$$



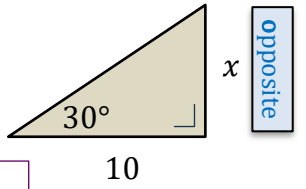
Pythagoras with Isosceles

$$x^2 = 3^2 + 3^2$$
$$x^2 = 9 + 9$$
$$x^2 = 18$$
$$x = \sqrt{18} \text{ or } 3\sqrt{2} \text{ or } 4.24$$



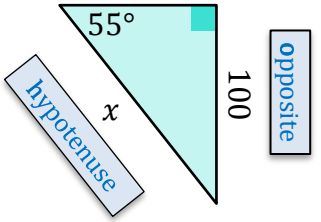
Using Ratios to Find Missing Sides

$$\tan 30° = \frac{x}{10}$$
$$x = 10 \tan 30°$$
$$x = 5.77$$



When x is on the top multiply both sides by the bottom of the fraction.

$$\sin 55° = \frac{100}{x}$$
$$x = \frac{100}{\sin 55°}$$
$$x = 122.08$$
$$x = 122 \text{ m}$$

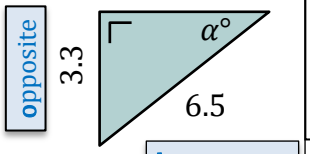


When x is on the bottom SWAP and divide.

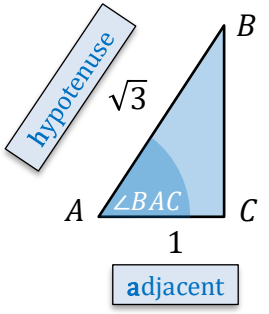
Using Ratios to Find Missing Angles

$$\sin \alpha° = \frac{3.3}{6.5}$$
$$\alpha° = \sin^{-1}\left(\frac{3.3}{6.5}\right)$$
$$\alpha° = 30.51$$
$$\alpha° \approx 31°$$

Press SHIFT sin



$$\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos \angle BAC = \frac{1}{\sqrt{3}}$$
$$\angle BAC = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$
$$\angle BAC = 54.73°$$

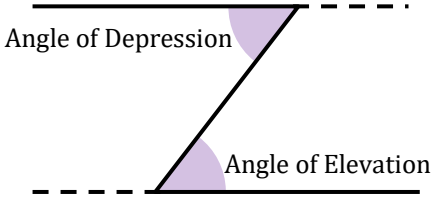


Tools To Measure With

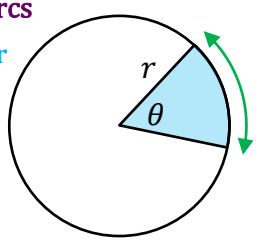
Clinometer for angles of elevation and depression. We can use a **trundle wheel** (or a measuring tape) to measure distance along the ground.

Alternate Angles (If lines parallel)

In the sketch the angles of **elevation** and **depression** are equal.



Sectors and Arcs



Area of Sector

$$\pi r^2 \times \frac{\theta}{360}$$

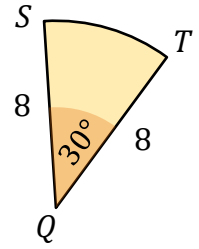
Length of Arc

$$2\pi r \times \frac{\theta}{360}$$

Find the length of the arc TS and the area of the sector SQT.

$$l = 2\pi r \times \frac{\theta}{360}$$
$$l = 2\pi(8) \times \frac{30}{360}$$
$$l = 4.2 \text{ m}$$

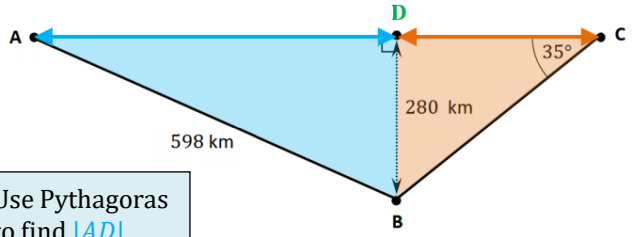
$$A = \pi r^2 \times \frac{\theta}{360}$$
$$A = \pi(8)^2 \times \frac{30}{360}$$
$$A = 16.8 \text{ m}^2$$



Multiple Triangles

In the diagram below, each of the points A, B, and C represents an airport. The plane travelled 598 km from Airport A to Airport B, and then travelled on to Airport C. Work out the distance |AC|, correct to the nearest km.

There are 2 right angles triangles here. We can use Pythagoras and the trigonometric ratios to find the distances |AD| and |DC|.



Use Pythagoras to find $|AD|$.

$$|AD|^2 + 280^2 = 598^2$$
$$|AD|^2 + 78400 = 357604$$
$$|AD|^2 = 279204$$
$$|AD| = \sqrt{279204}$$
$$|AD| = 528.4$$

$$|AC| = |AD| + |DC|$$
$$|AC| = 528.4 + 399.88$$
$$|AC| = 928.28$$
$$|AC| \approx 928 \text{ km}$$

Use Ratio to find $|DC|$

$$\tan 35° = \frac{280}{|DC|}$$
$$|DC| = \frac{280}{\tan 35°}$$
$$|DC| = 399.88$$

Part of the roof of a house is shown in the diagram.

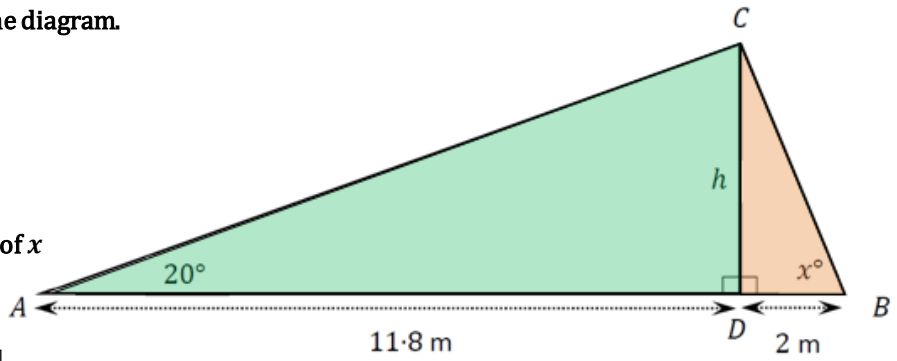
AB is perpendicular to DC.

$|AD| = 11.8$ m and $|DB| = 2$ m.

$|\angle CAD| = 20°$ and $|\angle DBC| = x°$.

Note: $\angle ACB$ is not a right angle.

Use trigonometry to work out the value of x to the nearest whole number.



We can use a trigonometric ratio with the right triangle on the left to find h.

$$\tan 20° = \frac{h}{11.8}$$
$$h = 11.8 \tan 20°$$
$$h = 4.29$$

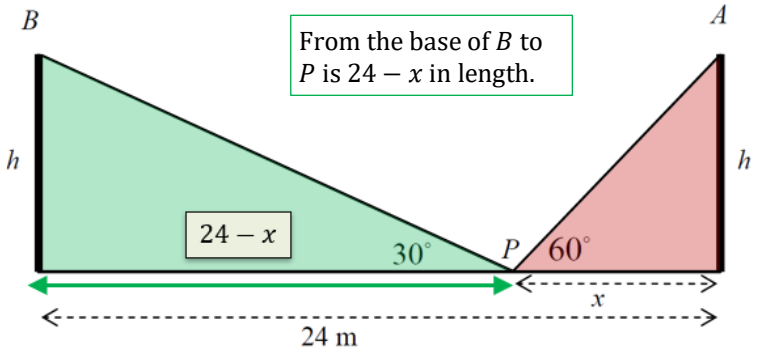
$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$

We can then use this value of h with the right triangle on the right to find the value of x.

$$\tan x = \frac{4.29}{2}$$
$$x = \tan^{-1}\left(\frac{4.29}{2}\right)$$
$$x = 65°$$

Press SHIFT to find an angle.

Two vertical poles A and B, each of height h, are standing on opposite sides of a level road. They are 24 m apart. The point P, on the road directly between the two poles, is a distance x from pole A. The angle of elevation from P to the top of pole A is 60°.



From the base of B to P is $24 - x$ in length.

Write h in terms of x using the triangle on the right.

$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 60 = \frac{h}{x}$$
$$h = x \tan 60$$
$$h = x(\sqrt{3})$$
$$h = \sqrt{3}x$$

Write h in terms of x using the left triangle.

$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 30° = \frac{h}{24 - x}$$
$$h = (24 - x) \tan 30$$
$$h = (24 - x)\left(\frac{\sqrt{3}}{3}\right)$$
$$h = \frac{(24 - x)\sqrt{3}}{3}$$

Equate the h's

$$\sqrt{3}x = \frac{(24 - x)\sqrt{3}}{3}$$
$$3\sqrt{3}x = (24 - x)\sqrt{3}$$
$$3x = 24 - x$$
$$3x + x = 24$$
$$4x = 24$$
$$x = 6$$

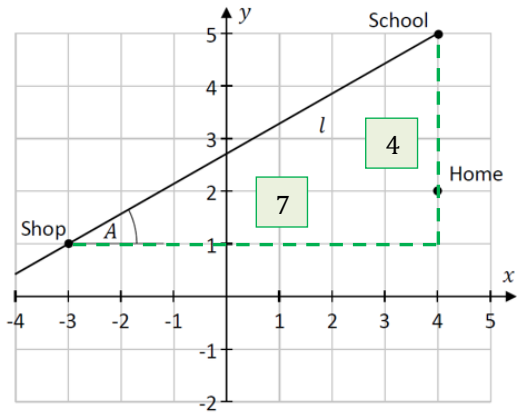
Find h

$$h = \sqrt{3}x$$
$$h = \sqrt{3}(6)$$
$$h = 6\sqrt{3}$$

Use trigonometry and the slope of l to work out the size of the angle A.

We can find the angle a line makes with the horizontal by finding the $\tan^{-1}(\text{slope})$.

$$\tan A = \frac{4}{7}$$
$$A = \tan^{-1}\left(\frac{4}{7}\right)$$
$$A = 29.7°$$

Press **SHIFT** to find angle.

$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{\text{rise}}{\text{run}}$$
$$= \text{slope}$$